FDTD Treatment of Partially Magnetized Ferrites With a New Permeability Tensor Model

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***Abstract*—This paper outlines the finite-difference time-domain (FDTD) treatment of partially magnetized ferrites characterized by a permeability tensor model, which was recently published [15]. Its causal aspect makes this tensor well adapted to time-domain simulations. Validation is demonstrated for a resonant ferrite structure. Numerical and analytical results are compared, showing good agreement.**

***Index Terms*—FDTD, ferrite.**

I. Introduction

A LARGE number of nonreciprocal devices such as isolators, circulators, or phase shifters include ferrites with an arbitrary magnetization state. Due to their complex geometry, ferrite devices do not generally accept analytical treatment. One solution consists of analyzing their behavior with numerical methods. The finite-difference time-domain (FDTD) method was already applied to electromagnetic problems, including ferrites. Two main approaches are used. The first approach is based on time-domain discretization of both Maxwell's curl equations and Gilbert's equation of motion [2]–[7], [13]. The second method consists of introducing the ferrite-material frequency characteristics in the FDTD algorithm after inverse Fourier transform and convolution [6], [8], [14]. This requires a causal permeability tensor μ.

When subjected to a magnetic field H_o, a ferrite becomes anisotropic and dispersive. If H_o is sufficiently large, the ferrite is saturated and its behavior is well described by a Polder tensor [1]. The μ-tensor is then causal, and FDTD treatment is coherent [8]. In case of low H_o, the ferrite is partially magnetized. Many permeability tensor models have been proposed [9]–[12], but were not available over a large frequency range and were not causal, contrary to the recently published model of Gelin–Berthou [15]. Pereda *et al.* [14] applied the FDTD method to partially magnetized ferrites characterized by the Green and Sandy [11] tensor model. As a result, the study was restricted to the lossless case because this tensor is only real (no loss) and, consequently, not causal. Moreover, Pereda *et al.* make the approximation that the permeability of the completely demagnetized ferrite is constant and equal to its average value over the frequency range. This approximation introduces a lack of accuracy, particularly for low frequencies.

In this paper, devices with nonsaturated ferrites are treated by the FDTD method and the model of Gelin and Berthou [15]. The causality of this model is verified, and we explain how to include it in the FDTD algorithm. This approach is validated on a resonant ferrite structure for which a modal study is possible.

II. Unsaturated Ferrite Treatment with FDTD

A. The Classical FDTD Method

The well-known FDTD algorithm introduced by Yee [16] consists of a discretization of the Maxwell's curl equations in time and space. The studied and surrounding space are decomposed into elementary parallelepipedic cells.

Electric and magnetic fields are computed, respectively, at the center of the edges and the faces of each cell, using the Yee scheme [16].

B. The FDTD Algorithm for an Unsaturated Ferrite

In the case of a ferrite, it is necessary to add equations which describe the anisotropic and dispersive behavior of the medium. When the ferrite is partially magnetized there is no differential equation (like Gilbert's equation in the saturated ferrite [2]–[7]) relating the magnetic field and magnetic moment.

This is the reason why we chose to characterize the ferrite using a permeability tensor $[\mu(\omega)]$ in frequency domain and took its inverse Fourier transform to return to the time domain. When the ferrite is magnetized in the O_z direction, the general expression of $[\mu(\omega)]$ is the following:

$$[\mu(\omega)] = \mu_o \begin{bmatrix} \mu_r(\omega) & -j\kappa(\omega) & 0 \\ j\kappa(\omega) & \mu_r(\omega) & 0 \\ 0 & 0 & \mu_z(\omega) \end{bmatrix} \tag{1}$$

with $\mu_r(\omega = 1 + \chi(\omega)$ where $\chi(\omega)$ is the magnetic susceptibility.

The expressions of each term of the permeability tensor will be presented later. In order to deal with the problem with the time-domain FDTD method, the time dependence of the permeability must be determined by calculating the inverse

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Fourier transform of $[\mu(t)]$

$$[\mu(t)] = \mu_o \begin{bmatrix} \mu_r(t) & \kappa(t) & 0 \\ -\kappa(t) & \mu_r(t) & 0 \\ 0 & 0 & \mu_z(t) \end{bmatrix} \tag{2}$$

with $\mu_r(t) = \delta(t) + \chi(t)$ where $\delta(t)$ is the Dirac distribution. Then $\vec{B}(\dagger)$ is calculated using convolution $[\mu(t)] * \vec{H}(\dagger)$

$$\begin{cases} B_x(t) = (\delta(t) + \chi(t)) * H_x(t) + \kappa(t) * H_y(t) & (3) \\ B_y(t) = -\kappa(t) * H_x(t) + (\delta(t) + \chi(t)) * H_y(t) & (4) \\ B_z(t) = \mu_z(t) * H_z(t). & (5) \end{cases}$$

Finally, the following equations are solved successively at each time step and in each spatial cell:

$$\begin{cases} \text{rot}\vec{E} = -\dfrac{\partial \vec{B}}{\partial t} & (6) \\ \vec{B}(t) = [\mu(t)] * \vec{H}(t) & (7) \\ \text{rot}\vec{H} = \varepsilon_o \varepsilon_r \dfrac{\partial \vec{E}}{\partial t}. & (8) \end{cases}$$

The discretization of (6) gives $\vec{B}$ at the time step $n\Delta t$ as a function of $\vec{B}$ and $\vec{E}$ at the previous time step. Equation (8) gives $\vec{E}$ at the time step $n\Delta t$ as a function of $\vec{E}$ and $\vec{H}$ at $(n-1)\ \Delta t$. The discretization of (7) allows the complete resolution of the problem. It expresses $\vec{B}$ as a function of $\vec{H}$. The computation of the convolution product has already been presented in detail. It is done *recursively* and, thus, does not increase the computation time too much. This theory has already been applied to the study of saturated ferrites [8] using the Polder expressions for each term of the tensor. In the unsaturated case, it is necessary to use a *causal tensor*. This is the reason why we use a new permeability tensor model, which will be discussed in Section III.

III. New Model for a Ferrite Permeability Tensor

The partially magnetized material is considered as a group of regions, which are themselves divided into domains. The magnetization mechanism is only a *rotational* one. Each independent region of the material is represented by the angle between its anisotropy field and the field applied. The global behavior is obtained from a spatial average over all the responses of the regions. Each local response is obtained by solving two coupled Gilbert's equations in two neighboring domains. If one knows the magnetization state of ferrite (M/Ms), its anisotropy field (H_a), and the damping term (α), it is also possible to derive each permeability tensor component from a self-consistent model [15].

The final form of the tensor components is then given by the following integrals:

$$\mu_r = 1 + 1/4 \int_0^\pi (B(\vartheta)\cos^2\theta_1 + D(\vartheta) + C(\vartheta) \cdot \sin\theta_1 \cos\theta_1)\sin\vartheta . d\vartheta = 1 + \chi \tag{9}$$

$$\kappa = \frac{j}{4} \int_0^\pi (A(\vartheta)\cos\theta_1 - F(\vartheta)\sin\theta_1 - E(\vartheta)\cos\theta_1) \cdot \sin\vartheta . d\vartheta \tag{10}$$

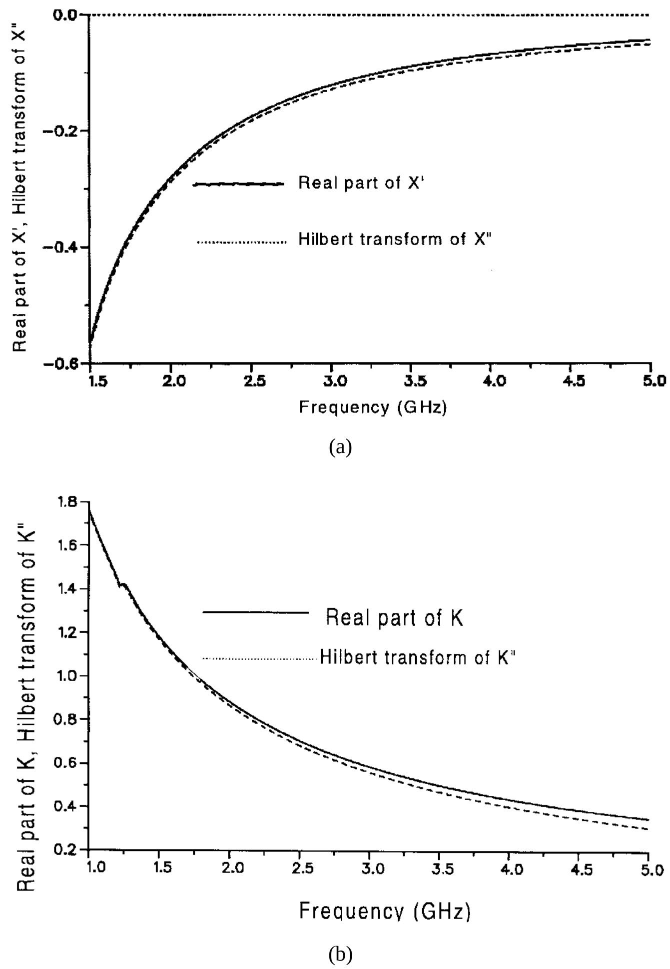

Fig. 1. Verification of causality. Comparison between $\chi'(f)$ and $-TH(\chi''(f))$, $(\kappa'(f))$, and $-TH(\kappa''(f))$ ($Ms = 71.6$ kA/m, $\text{Han} = 0.16$ kA/m, $M/Ms = 0.7$).

$$\mu_z = 1 + 1/2 \int_0^\pi (B(\vartheta)\sin^2\theta_1 - C(\vartheta)\sin\theta_1\cos\theta_1) \cdot \sin\vartheta . d\vartheta = 1 + \chi z \tag{11}$$

where $A(\vartheta)$, $B(\vartheta)$, $C(\vartheta)$, $D(\vartheta)$, $E(\vartheta)$, and $F(\vartheta)$ depend on ferrite characteristics (anisotropy field, magnetization state M/Ms, damping term α, etc.).

The next step is to check the causality of the model before including it in time-domain electromagnetic methods such as FDTD. The integrations seem to make this check difficult. By definition, causality is proved when the Hilbert transform of the imaginary part is exactly the opposite of the real part, and when the Hilbert transform of the real part is the imaginary part.

For example, in the frequency domain, it is possible to write: $\mu_r = \mu_r' - j\mu_r'' = 1 + \chi' - j\chi''$, $\kappa = \kappa' - j\kappa''$.

The causality of μ and κ is proven if

$$\chi'(f) = -TH(\chi''(f)) \quad \kappa'(f) = TH(\kappa''(f))$$

and

$$\chi''(f) = TH(\chi'(f)) \quad \kappa''(f) = -TH(\kappa'(f)). \tag{12}$$

We used a mathematical algorithm to perform the Hilbert transforms. Fig. 1 shows the results which imply the causality

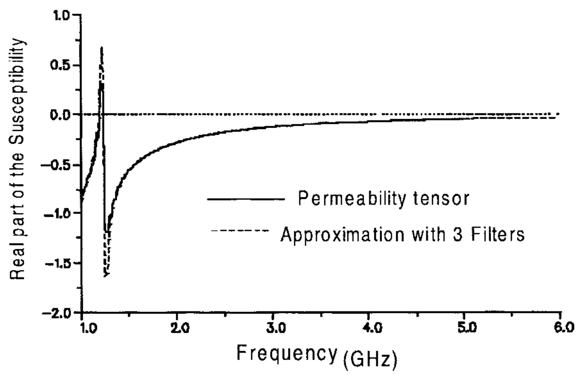

Fig. 2. Approximation of $\chi(f)$ by a sum of three filters. $C_1 = 1.023\ 10^8$, $\omega_{01} = 1.250\ 10^9$, $\alpha = 5\ 10^{-3}$. $C_2 = 6.608\ 10^9$, $\omega_{02} = 1.514\ 10^8$, $\alpha = 5\ 10^{-3}$. $C_3 = 3.361\ 10^7$, $\omega_{03} = 1.266\ 10^9$, $\alpha = 5\ 10^{-3}$. ($Ms = 71.6$ kA/m, $\mathrm{Han} = 0.16$ kA/m, $M/Ms = 0.7$.)

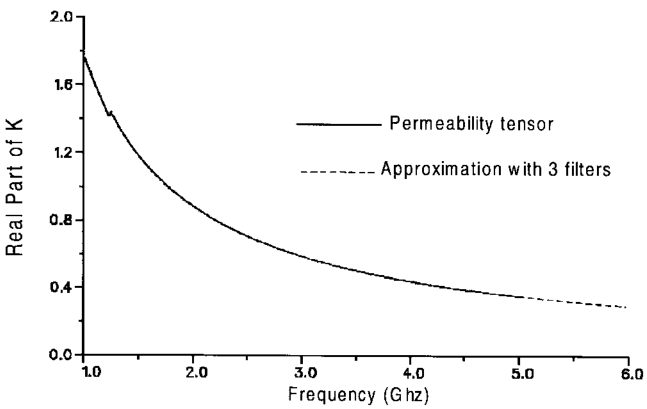

Fig. 3. Approximation of $\kappa'(f)$ by a sum of three filters. $C_1 = 1.763\ 10^9$, $\omega_{01} = 7.436\ 10^7$, $\alpha = 5\ 10^{-3}$. $C_2 = 1.853\ 10^6$, $\omega_{02} = 1.249\ 10^9$, $\alpha = 5\ 10^{-3}$. $C_3 = 2.623\ 10^5$, $\omega_{03} = 1.279\ 10^9$, $\alpha = 5\ 10^{-3}$. ($Ms = 71.6$ kA/m, $\mathrm{Han} = 0.16$ kA/m, $M/Ms = 0.7$.)

of each tensor component. It is then possible to use the new tensor model in the FDTD algorithm previously discussed.

The last problem concerns the calculation of the inverse Fourier transform. It is impossible to use an analytical calculation because of the integration over ϑ in the tensor elements. We avoid this problem by approaching each tensor element with a sum of digital second-order filters. This approximation is done numerically using MATLAB. The following two different filters are used to approximate μ and k (by analogy with Polder's formulas):

$$\mu_r(\omega) = 1 + \sum C_i \frac{\omega_{oi} + j\omega\alpha}{((\omega_{oi} + j\omega\alpha)^2 - \omega^2)} \tag{13}$$

$$\kappa(\omega) = \sum C'_i \frac{\omega}{((\omega'_{oi} + j\omega\alpha)^2 - \omega^2)} \tag{14}$$

where ω_{oi} and ω'_{oi} are the central frequencies of filter number i, C_i and C'_i the weighting constant of i-filter, and α the damping term. Then, the inverse Fourier transform is given by

$$\mu_r(t) = \delta(t) + \sum \frac{C_i}{\omega_{oi}} \nu_{oi} e^{-\alpha\nu_{oi}t} \cdot (\alpha \cos(\nu_{oi}t) + \sin(\nu_{oi}t)) \cdot u(t) \tag{15}$$

$$\kappa(t) = \sum \frac{C'_i}{\omega'_{oi}} \nu'_{oi} e^{-\alpha\nu'_{oi}t} \cdot (\cos(\nu'_{oi}t) - \alpha \sin(\nu'_{oi}t)) \cdot u(t) \tag{16}$$

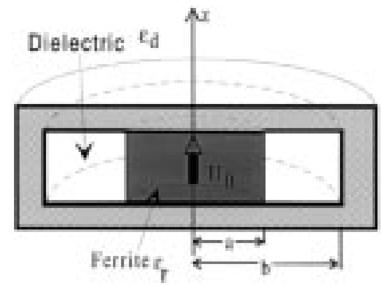

Fig. 4. A cylindrical ferrite resonator centered in a metallic cavity.

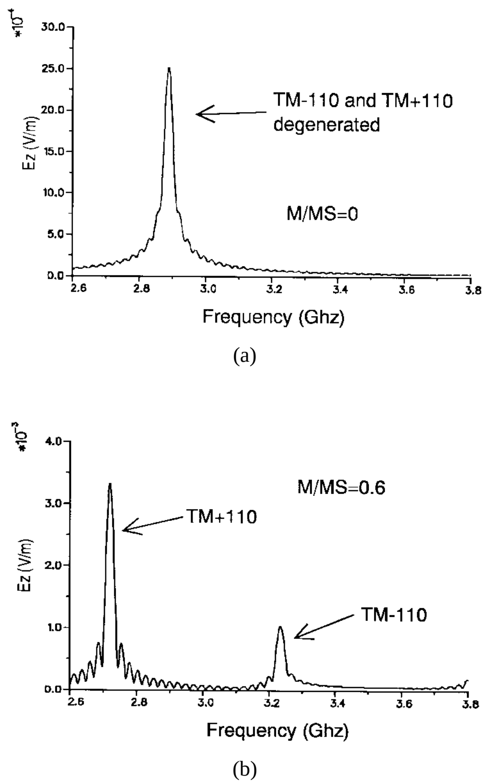

Fig. 5. Resonant frequencies of modes TM $+/-$ 110 for different values of M/Ms. $a = 13$ mm, $b = 26$ mm, $\varepsilon_d = 1$, $\varepsilon_f = 14$. FDTD cells: $\Delta x = \Delta y = 0.65$ mm, $\Delta t = 5.73\ 10^{-12}$.

where $\delta(t)$ is the Dirac distribution, $u(t)$ the unit step function, $\nu_{oi} = \omega_{oi}/(1+\alpha^2)$ and $\nu'_{oi} = \omega'_{oi}/(1+\alpha^2)$.

Some examples of numerical approximations are presented in Fig. 2 [for $\mu(\omega)$] and Fig. 3 [for $\kappa(\omega)$]. There appears to be good agreement between the approximations and the curves given by the tensor formulas. Only three filters are used here. It is interesting to point out that agreement can be improved (particularly near resonances) using a greater number of filters. Another way to improve the approximation near resonance is to allow different values of α for each filter. This method could be particularly interesting for low values of the ratio M/Ms when several resonances appear in the evolution of μ and κ versus frequency.

The causality of these filters allows their inverse Fourier transform to be calculated, and then to compute the global FDTD algorithm. Results are presented Section IV.

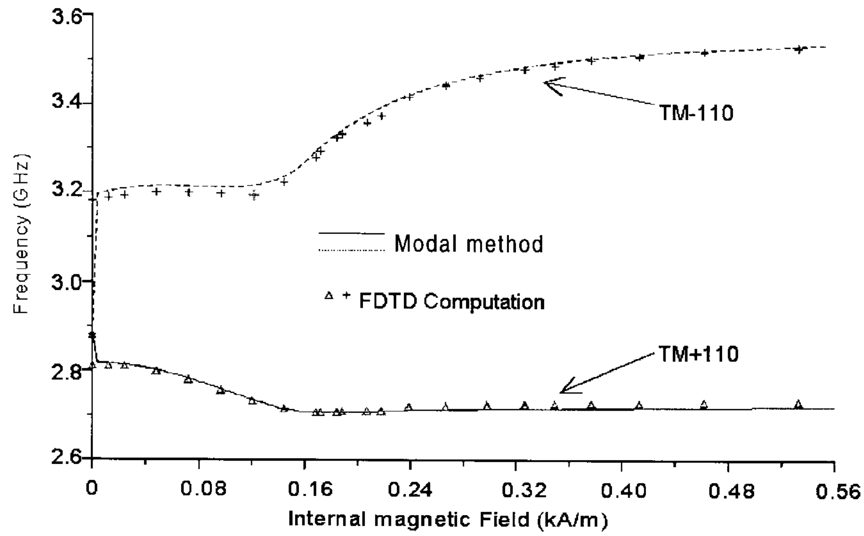

Fig. 6. Evolution of resonant frequencies of TM +/− 110 modes versus the magnetic field. Comparison between the modal method and FDTD computation ($Ms = 71.6$ kA/m, $\mathrm{Han} = 0.16$ kA/m). $a = 13$ mm, $b = 26$ mm, $\varepsilon_d = 1$, $\varepsilon_f = 14$. FDTD cells: $\Delta x = \Delta y = 0.65$ mm, $\Delta t = 5.73\ 10^{-12}$.

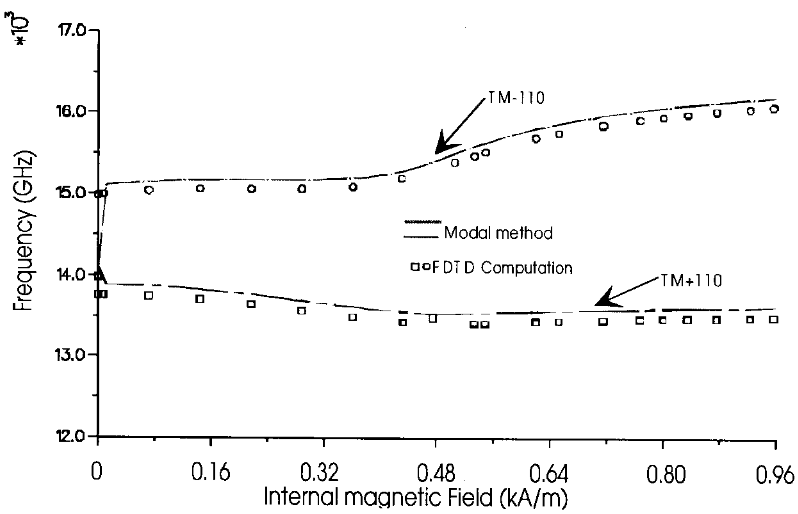

Fig. 7. Evolution of resonant frequencies of TM +/− 110 modes versus the magnetic field. Comparison between the modal method and FDTD computation ($Ms = 292.6$ kA/m, $\mathrm{Han} = 0.48$ kA/m). $a = 8$ mm, $b = 16$ mm, $\varepsilon_d = 1$, $\varepsilon_f = 16$. FDTD cells: $\Delta x = \Delta y = 0.40$ mm, $\Delta t = 3.771\ 10^{-12}$.

IV. Application to a Resonant Ferrite Structure

To show the validity of this new formulation, resonance frequencies of a cylindrical cavity where a cylindrical ferrite is centered (see Fig. 4) are calculated. A two-dimensional (2-D) FDTD calculation is used because $\mathrm{TM} + 110$ and $\mathrm{TM} - 110$ are two modes that do not depend on the height. Their resonant frequencies are calculated for different values of the bias magnetic field H_o. The computation is performed by the FDTD method directly in the time domain, and resonant frequencies are obtained after using a Fourier transform. The results obtained by 2-D FDTD are compared to those given by the mode-matching method [17].

Fig. 5 shows the resonance frequencies of $\mathrm{TM} + 110$ for two values of H_o ($H_o = 0$ kA/m and $H_o = 0.14$ kA/m). The FDTD computation shows that the two modes are degenerated when the ferrite is completely demagnetized. When H_o increases, the resonance frequency of $\mathrm{TM} + 110$ increases and the $\mathrm{TM} - 110$ one decreases. This is a well-known phenomenon for unsaturated ferrite.

Figs. 6 and 7 show the resonances of $\mathrm{TM} + 110$ versus H_o for two different ferrites ($4\pi\ Ms = 71.6$ kA/m and $4\pi\ Ms = 292.6$ kA/m) and for two cavity sizes. The computation is obtained by using both the FDTD and mode-matching methods. Good agreement appears between the curves, which validates our new approach for unsaturated ferrite. The modal method has already been validated by a comparison with experiment [17], this is why no experimental results are presented in this paper.

V. Conclusion

A new method has been presented in this paper for the study of partially magnetized ferrite using the FDTD method. It uses a new permeability tensor recently developed by Gelin and Berthou. The study has been validated by the computation of resonance frequencies of a 2-D resonant structure. This method requires two additional storage variables (for each filter) comparing to the "classical" FDTD algorithm based on the Yee's scheme [2] and the resolution of Maxwell's curl equations and Gilbert's equation. These variables are necessary to compute the convolutions products of (3) and (4).

The extension of this method to model three-dimensional (3-D) devices like ferrite antennas or ferrite circulators is currently under consideration.

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